Dynamic Modelling of Multi-GHz Travelling Wave Particle Deflectors using the Boundary Element Method and Quasi-TEM Coupled Line Theory

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Most broadband multi-gigahertz charged particle deflection systems are travelling wave structures either in the form of single or double-sided meanderlines or helices. These structures allow a velocity match of the deflecting signal to the particle beam and offer greatly improved bandwidth and sensitivity over solid plate deflectors of equal size.

A computer model has been developed as a design tool to calculate the full time-varying 3-dimensional fields for deflection structures having multiple conductors of arbitrary cross-section in the plane of deflection and uniform in the direction normal to the plane of deflection. The model uses a hybrid analytical method which combines solutions of the Quasi-TEM Coupled Lines Equations with the 2-dimensional Boundary Element Method (BEM) static field solutions to produce both a realistic circuit model and field components for calculation of beam deflection and aberrations.

Both a meanderline deflector, and a helix appropriately cut and unrolled (Fig 1), can be viewed as a set of N coupled transmission lines with specific boundary conditions at their ends and one or more reference ground planes. Under quasi-TEM assumptions of zero E and B components in the direction of propagation, the time-varying electric field in such a system can be derived from a time-varying scalar potential ϕ:

$$\vec{E}(\vec{r},t) = -\nabla\phi(\vec{r},t) \qquad \vec{B}(\vec{r},t) = -\int \nabla \times \vec{E}(\vec{r},t)dt$$

$$\phi(\vec{r},t) = \sum_{i=1}^{N} F_i(y,z)V_i(x,t) \qquad V_i(x,t) = A_i e^{j(\omega t - kx)} + B_i e^{j(\omega t + kx)}$$

The scalar potential (Fig. 2) is a superposition of forward and backward propagating 2-D potentials $F_i(y,z)$ with amplitudes A_i and B_i determined by the voltage wave on each line. The individual $F_i(y,z)$ form a BEM unit basis set of potential solutions for the structure and are derived by setting the voltage on the i^{th} conductor to 1 and to 0 on all other conductors.The line voltages and currents V and I, given by integrals in the y-z plane, form a set of vectors which are governed by the Coupled Line (Telegraphers) equations:

$$\frac{\partial V}{\partial x} = -[L]\frac{\partial I}{\partial t} \qquad \frac{\partial I}{\partial x} = -[C]\frac{\partial V}{\partial t}$$

where [L] and [C] are the inductance and capacitance matrices. The voltage and current solutions of the Coupled Line equations can be used to calculate the complete frequency domain network parameters (S, H,Z, Y, etc.) of the deflector as a circuit element (Fig. 3). By an inverse fourier transform, the time domain step response is computed (Fig. 4). The circuit impedance, transmission and reflections are then readily displayed.

The BEM technique is particularly convenient in providing the capacitance matrix [C], and for resetting the charge densities, as dictated by the the voltage waves $V_i(x,t)$, at each instant of time during integration of the equations of motion.

In summary, the modelling tool allows one to study such questions as: impedance matching to the signal source, propagation velocity versus frequency, deflection sensitivity rolloff versus frequency, high frequency spot abberations, position tolerancing, etc. This method has been used to explore the performance limits of structures with as many as 20 conductors.

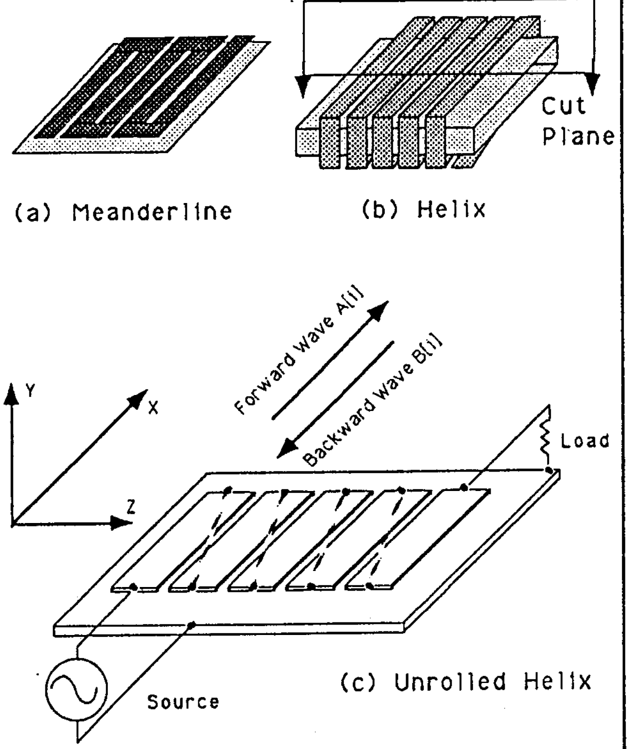
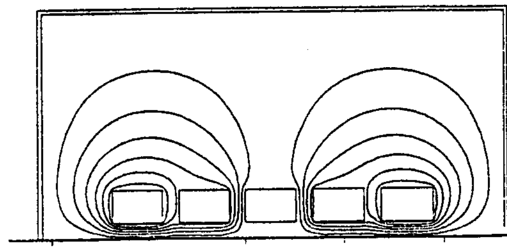
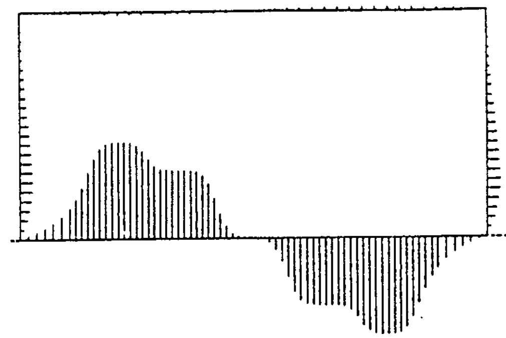


Fig. 1

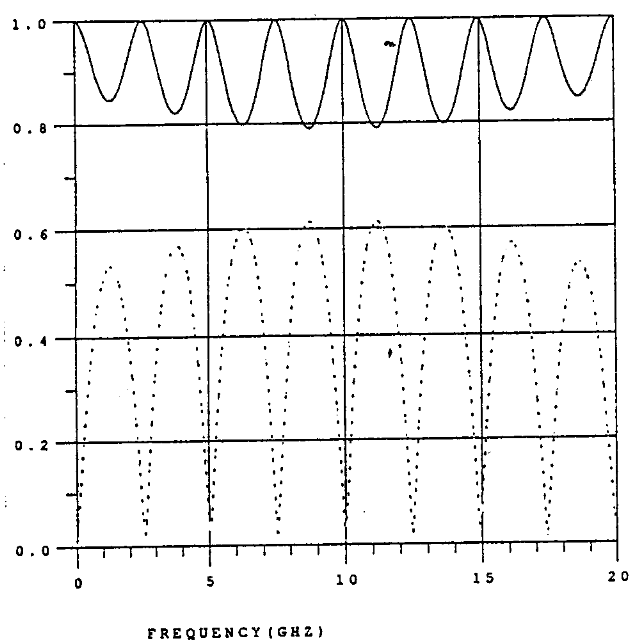


(a) Instantaneous scalar potential for a simple 5 "turn" helix driven by a sinewave of 1/2 wavelength=4 turns.



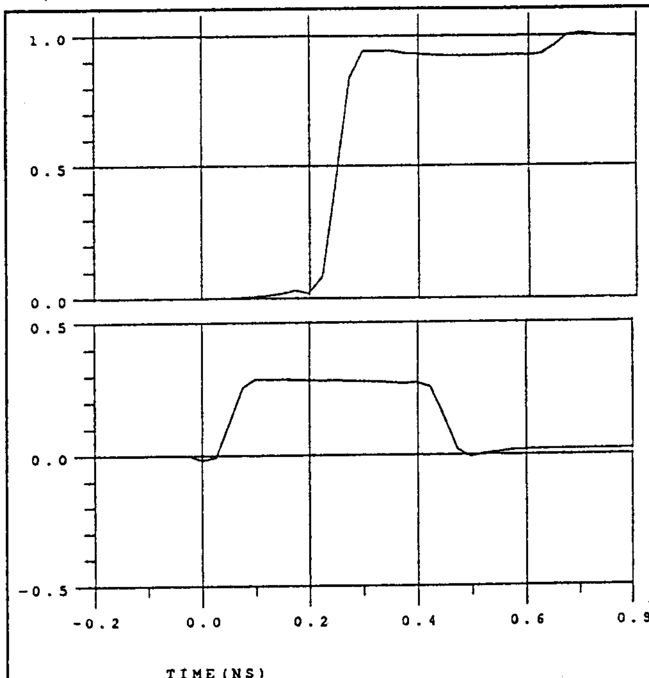
(b) Instantaneous charge densities on surrounding ground plane. Density is given by length of lines.

Fig. 2



Frequency domain S-parameters for 5-turn helix of Fig. 2. Top curve is |S21| and bottom curve is |S11|.

Fig. 3



Time domain transmission (top curve) and reflection (bottom curve) step responses for 5-turn helix.

Fig. 4